Bot. Bull. Academia Sinica **13**: 37-45 (1972)

A MONTE CARLO SIMULATION ON HOST-PARASITE INTERACTION

I. Random Spacial Pattern[1]

TSAN-LONG LIN[2]

Abstract

A Monte Carlo simulation is made for host-parasite interaction in the crop field. The results showed that the model could represent the economic law of decreasing returns. It also revealed a way of describing density interdependence between the parasite released and the host in the field, which would enable us to reach an optimum decision on biological control policy for the crop pest. Considerable variations attributable to the sampling existed thus a further study on the estimation of unknown parameters from actual field experiments remains to be made.

1. Introduction

Since the works of Lotka (1925) and Voltera (1931), much mathematical analyses of interaction between two or more species have been made (Moran 1950, Pearce 1970, and Samuelson 1971) with respect to deterministics. Comparatively little have been done however on the corresponding stochastic model (Bartlett 1960 and Pielou 1970) due to non-linearlity of the transition probabilities (Bailey 1963). Application of the models in designing a pest control program is thus greatly hindered. Present investigation is concerned with a model of the spread of the parasite in the field after release and a way of describing host-parasite interaction when the spacial pattern of the pest is random, i.e., Poisson type. It is assumed here that the host is an agriculture pest and the parasite is a parasitic wasp.

2. Statistical model

Suppose that a parasite is released at a point, say O, of a crop field, and after prescribed lapse of time a survey is made to determine the position at which the parasite is found. Let $P(r)$ be the probability of finding the parasite within the limits of a circle with a radius r centred at O. Assuming the

(1) This work was supported by National Science Council, Rep. of China.

(2) Associate Professor, Department of Agronomy, College of Agriculture, National Taiwan University.

movement of the parasite over the field is random and the probability of finding the parasite proportional to the area of the survey made, following relationships can be perceived:

$$P(r+\Delta r)=P(r)+\lambda[(r+\Delta r)^2-r^2][1-P(r)]+O(\Delta r) \quad (1)$$

where λ is an unknown parameter.

If Δr approaches zero, we see that

$$\frac{dP(r)}{dr}=2\lambda r[1-P(r)] \quad (2)$$

Solving (2), we obtain

$$P(r)=1-e^{-\lambda r^2}$$

If the parasite moves without polarity and is not affected by wind, the probability of finding it in the fan-shaped section of radian θ is given by

$$F(\theta,r)=\frac{\theta}{2\pi}P(r)=\frac{\theta}{2\pi}(1-e^{-\lambda r^2}) \quad (3)$$

or in the form of probability density function (p. d. f.)

$$f(\theta,r)=\frac{\partial^2 F(\theta,r)}{\partial\theta\partial r}=\frac{\lambda r e^{-\lambda r^2}}{\pi} \quad (4)$$

Converting the coordinate system into rectangular one, we obtain

$$f(x,y)=\frac{\lambda}{\pi}e^{-\lambda(x^2+y^2)} \quad (5)$$

which is a form of two dimensional normal distribution with one unknown parameter λ, which may be called "diffusion parameter".

If N_0 individuals of the parasite released at O move independently, the probability of finding N individuals in a specific rectangular plot with d and g as its length and width is given by

$$P_N(x,y|d,g)=\binom{N_0}{N}[F(x,y|d,g)]^N[1-F(x,y|d,g)]^{N_0-N} \quad (6)$$

where

$$F(x,y|d,g)=\frac{\lambda}{\pi}\int_x^{x+d}\int_y^{y+g}e^{-\lambda(x^2+y^2)}dx\,dy \quad (7)$$

and (x,y) is the coordinate of the lower left hand corner of the plot.

As it is expected from the property of the normal distribution, Table 1 displays that the probability defined by (7) decreases rapidly with the distance from the origin O.

Now suppose that the size of the plot is small enough so that if the parasite and the host are both present, the probability of establishing parasism

Table 1. *Probabilities of Finding the Parasite after release with $d=1.25$ and $g=4.00$*

x	y	0.0	4.0	8.0	12.0
0.020	0.00	0.0284	0.0155	0.0046	0.0007
	1.25	0.0267	0.0146	0.0003	0.0001
	2.50	0.0236	0.0129	0.0038	0.0006
	3.75	0.0196	0.0107	0.0032	0.0005
	5.00	0.0153	0.0083	0.0025	0.0003
	6.25	0.0112	0.0061	0.0018	0.0003
	7.50	0.0077	0.0042	0.0012	0.0002
	8.75	0.0050	0.0027	0.0008	0.0001
	10.00	0.0030	0.0016	0.0005	0.0001
0.040	0.00	0.0513	0.0162	0.0016	0.0000
	1.25	0.0453	0.0143	0.0014	0
	2.50	0.0354	0.0112	0.0011	0
	3.75	0.0244	0.0077	0.0007	0
	5.00	0.0149	0.0047	0.0005	0
	6.25	0.0080	0.0025	0.0002	0
	7.50	0.0038	0.0012	0.0001	0
	8.75	0.0016	0.0005	0.0000	0
	10.00	0.0006	0.0002	0.0000	0

x	y	0.0	4.0	8.0
0.15	0.00	0.1230	0.0036	0.0000
	1.25	0.0784	0.0023	0
	2.50	0.0318	0.0009	0
	3.75	0.0083	0.0002	0
	5.00	0.0013	0.0000	0
	6.25	0.0001	0	0
	7.50	0.0000	0	0
2.5	0.00	0.1551	0.0007	0.0000
	1.25	0.0746	0.0003	0
	2.50	0.0172	0.0001	0
	3.75	0.0019	0.0000	0
	5.00	0.0001	0	0
3.5	0.00	0.1579	0.0001	0.0000
	1.25	0.0647	0.0000	0
	2.50	0.0087	0	0
	3.75	0.0004	0	0
	5.00	0.0001	0	0

is close to one. Then, if the parasite produces only one effective egg, the expected number of host killed will be

$$E[\min(X, Y)] = N_k \tag{8}$$

where X and Y are the number of hosts and parasites in a plot respectively. Clearly, N_k will depend on: (a) N_0, the number of the parasite released, (b) density and spatial pattern of the host, (c) number of points at which the parasite is released, and (d) the magnitude of λ which partly reflects the lapse of time after the release of the parasite.

The number of hosts in a plot is assumed to be distributed with the probability law:

$$P_x = \frac{e^{-\mu}\mu^x}{x!} \tag{9}$$

where P_x is the probability of a plot containing x individuals of the host, and μ is the mean number of host per plot.

3. The Experiments

Treatments considered are:

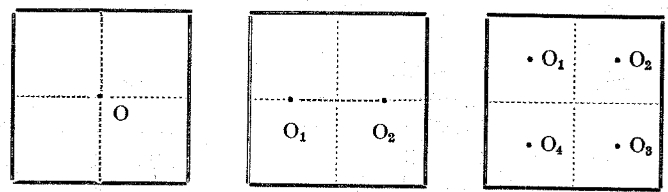
(a) Mean density of the host population

μ: 0.2, 0.4, 0.6, 0.8

(b) Number of parasites released

Number of points at which parasites are released	Number of parasites released at each point						
1	200	400	600	800			
2	100	200	300	400			
4	50	100	150	200	250	300	350

(c) Location of the points at which parasites are released



One point Two points Four points

(d) Defusion parameter

$\lambda=0.3,\ 0.5,\ 0.7$

Combinations of 177 are simulated on a CDC3150 computer, each with twenty replications. A square field with 20×20 plots is assumed in the simulation.

The data are generated on the computer as follows:

(a) Cumulative probabilities

$$P_r\{X\leqslant x\}=\sum_{j=0}^{x}\frac{e^{-\mu}\mu^{j}}{j!}$$

(b) Four hundred pseudo-random numbers $\{U_i\}$ in [0,1] are generated by means of congruence method

$$U_i=23\cdot U_{i-1} \bmod (10^8+1) \qquad U_0=47{,}594{,}118$$

which does not repeat until 5.8×10^6 terms. These uniform random numbers are then converted into Poisson random variables $\{x_i\}$ by the following inequality

$$P_r\{X\leqslant x_{i-1}\}<U_i\leqslant P_r\{X\leqslant x_i\}$$

The sequence $\{x_i\}$ obtained as above represents the number of hosts in each plot.

(c) N_0 pairs of uniform random Numbers $\{U_{1,i}, U_{2,i}\}$ in (0, 1) are generated. Using equation (3), two random variables θ_1 and r_1 are calculated as follows:

$$\theta_l = 2\pi U_{1,l}$$

$$r_l = \sqrt{-\frac{1}{\lambda}\log(1 - U_{2,l})}$$

These two random variables are then substituted into the equations

$$x_l = r_l \cos\theta, \qquad y_l = r_l \sin\theta$$

to give the coordinates of the plot at which the l-th parasite is found. The rectangular coordinate system which identifies the position of each plot is taken in the way that the origin of the system is placed at the point at which the parasite is released and the positive X-axis is directed to the east.

(d) If l-th plot contains x_l individuals of the hosts and y_l individuals of the parasites, minimum of (x_l, y_l), or z_l, is taken to be the number of hosts killed, by the assumption previously given. The total number of pests killed will then be

$$N_k = \sum_{l=1}^{400} z_l$$

(e) The processes described above constitute an experiment for a specific combination of (λ, μ). Twenty replications are made for each combination.

4. Results and Discussion

Results are shown in Tables 2, 3 and 4. They suggest that the model developed here represents the following facts:

(a) The model represents the well known economic law that the marginal return will decrease as the input is increased. The fact can be seen from Tables 2 and 3. This means that the technique can be employed to determine the most profitable number of parasites to be released if the average cost of the release and the crop damage by the host are known. For estimating unknown parameters, well designed field experiments are essential.

(b) The model also indicates that the marginal increases in parasitism by increasing the points of release is greater than by increasing the number of parasites. This suggests that the optimum policy might be the function of at least four variables, namely, the activity of the parasite λ, the density of hosts μ, the cost for maintaining the parasite and labour cost for releasing the parasite at the crop field.

(c) The subsequent statistical problem, little of which has been done, is how to estimate the unknown parameters from the actual field experiments. For instance, the interaction which might be present between the released parasites and those of natural population, and the sampling variation which

Table 2. *Results of Simulation—Number of Hosts Killed*

λ	μ	Number of Hosts	Number of Points of Release	200	400	600	800	1,000	1,200	1,400
				Number of Parasites Released						
0.30	0.20	85.05	1	8.60	9.70	10.55	11.05			
			2	11.80	15.45	18.40	19.05			
			4	18.10	24.35	27.55	30.30	30.33	30.30	32.00
0.30	0.40	164.45	1	14.05	16.25	18.70	19.20			
			2	22.05	27.65	31.35	33.05			
			4	33.95	47.45	55.75	60.55	62.70	66.20	68.60
0.30	0.60	246.00	1	22.25	25.75	29.30	30.80			
			2	34.35	43.70	49.50	51.70			
			4	50.65	70.70	78.00	86.95	86.10	90.70	93.25
0.30	0.80	321.35	1	28.80	35.35	39.65	36.95			
			2	41.45	54.65	62.10	65.20			
			4	60.60	85.15	101.60	108.00	118.15	122.60	130.35
0.50	0.20	80.10	1	5.75	6.95	7.65	8.10			
			2	8.55	10.55	11.65	12.55			
			4	12.30	16.55	18.40	19.85	22.90	23.85	24.55
0.50	0.40	157.90	1	10.15	12.60	13.75	14.20			
			2	16.45	20.40	22.15	23.25			
			4	26.20	34.25	38.60	41.35	40.40	42.70	43.95
0.50	0.60	240.05	1	15.25	18.00	19.75	20 75			
			2	24.25	29.25	33.05	35.75			
			4	36.20	47.25	54.20	57.20	64.40	67.35	68.90
0.50	0.80	307.20	1	18.70	21.55	22.40	23.95			
			2	30.05	38.05	41.20	45.10			
			4	46.10	61.25	69.20	74.40	81.85	84.85	88.75
0.70	0.20	77.20	1	4.30	4.40	4.60	5.25			
			2	7.05	7.65	8.85	9.55			
			4	11.40	13.40	15.15	15.50	12.80	17.60	18.45
0.70	0.40	161.30	1	8.30	9.80	10.65	10.73			
			2	13.80	16.50	17.90	18.25			
			4	21.95	27.55	30.35	31.60	34.30	34.80	36.20
0.70	0.60	245.15	1	10.85	12.90	13.80	14.40			
			2	19.05	22.30	26.50	27.35			
			4	31.75	38.80	45.35	47.30	49.80	52.70	54.20
0.70	0.80	318.40	1	15.50	19.05	20.45	22.15			
			2	25.80	31.20	34.00	36.05			
			4	38.95	52.50	57.55	61.45			

λ: Diffusion Parameter of Parasite.

μ: Mean Density of Host.

Table 3. *Marginal Increase of Parasitism for an Additional Increase of the Parasite*

λ	μ	Number of Hosts	Number of Points of Release	Number of Parasites Released: 0–200	200–400	400–600	600–800	800–1,000	1,000–1,200	1,200–1,400
0.30	0.20	85.05	1	0.04300	0.00550	0.00400	0.00250			
			2	0.05900	0.01825	0.01475	0.00325			
			4	0.09050	0.08125	0.01600	0.01225	0.00000	0.00850	0.00000
0.30	0.40	164.45	1	0.07025	0.01100	0.01225	0.00250			
			2	0.11025	0.02800	0.01850	0.00850			
			4	0.16975	0.06750	0.04150	0.02400	0.01075	0.01750	0.01200
0.30	0.60	246.00	1	0.11125	0.01750	0.01775	0.00750			
			2	0.17225	0.04625	0.02900	0.01100			
			4	0.25325	0.10025	0.03650	0.04475	-0.00425	0.02300	0.01275
0.30	0.80	321.35	1	0.14400	0.03275	0.02150	0.00150			
			2	0.20725	0.06600	0.03725	0.01550			
			4	0.30300	0.12275	0.08225	0.03200	0.05075	0.02225	0.03875
0.50	0.20	80.10	1	0.02875	0.00600	0.00350	0.00215			
			2	0.04275	0.01000	0.00550	0.00450			
			4	0.06150	0.02120	0.00925	0.00725	0.01525	0.00475	0.00350
0.50	0.40	157.90	1	0.05075	0.01225	0.00575	0.00225			
			2	0.08225	0.01975	0.00875	0.00550			
			4	0.13100	0.04025	0.02175	0.01375	-0.00475	0.01150	0.00625
0.50	0.60	240.05	1	0.07625	0.01375	0.00875	0.00500			
			2	0.12125	0.02500	0.01900	0.01350			
			4	0.18100	0.05525	0.03475	0.01500	0.03600	0.01465	0.00775
0.50	0.80	307.20	1	0.09350	0.01425	0.00425	0.00750			
			2	0.15025	0.04000	0.01575	0.01950			
			4	0.23050	0.07525	0.03975	0.02600	0.03725	0.01500	0.01950
0.70	0.20	77.20	1	0.02150	0.00050	0.00100	0.00425			
			2	0.03525	0.00300	0.00600	0.00350			
			4	0.05700	0.01000	0.00875	0.00175	-0.01350	0.02400	0.00425
0.70	0.40	161.30	1	0.04150	0.00700	0.00400	0.00050			
			2	0.06900	0.01350	0.00700	0.00175			
			4	0.10975	0.02800	0.01500	0.00525	0.01350	0.00250	0.00700
0.70	0.60	245.15	1	0.05425	0.01025	0.00400	0.00300			
			2	0.09525	0.01625	0.02100	0.00425			
			4	0.15875	0.03525	0.03275	0.00975	0.00750	0.01450	0.00750
0.70	0.80	318.40	1	0.07750	0.01775	0.00725	0.00850			
			2	0.12900	0.02700	0.01400	0.01025			
			4	0.19475	0.06775	0.02525	0.01950			

Table 4. *Ranges of Variation for Twenty Replications*

λ	μ	Number of Points of Release	Pests		200		400		600		800		1,000		1,200		1,400	
			Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
		1	102	70	12	5	14	5	14	6	15	6						
0.30	0.20	2	102	70	20	5	23	9	25	12	29	12						
		4	102	70	27	11	33	17	37	19	39	19	39	18	41	18	41	24
		1	202	136	23	7	26	8	33	9	28	9						
0.30	0.40	2	202	136	34	14	37	19	44	20	44	23						
		4	202	136	45	24	58	36	69	40	82	42	53	12	83	52	79	58
		1	278	221	34	14	33	19	42	23	43	24						
0.30	0.60	2	278	221	44	29	59	36	65	39	64	41						
		4	278	221	73	42	80	57	92	62	103	72	105	75	104	61	116	64
		1	349	295	43	19	49	28	60	30	55	33						
0.30	0.80	2	349	295	51	31	69	50	71	48	79	47						
		4	349	295	78	49	110	64	125	81	137	90	126	106	132	115	141	118
		1	102	60	11	1	14	1	14	2	15	1						
0.50	0.20	2	102	60	18	3	16	2	22	4	20	6						
		4	102	60	19	7	24	10	28	11	28	11	35	16	31	16	30	16
		1	186	136	15	3	18	5	22	8	21	7						
0.50	0.40	2	186	136	25	9	28	11	29	10	30	16						
		4	186	136	37	16	43	25	55	30	54	31	48	28	55	30	51	33
		1	270	209	25	7	25	8	28	12	29	12						
0.50	0.60	2	270	209	35	15	43	17	43	24	47	24						
		4	270	209	45	25	66	39	68	41	73	45	75	52	86	53	81	54
		1	348	273	26	10	28	14	28	14	31	13						
0.50	0.80	2	348	273	43	23	51	25	58	25	63	31						
		4	348	273	57	39	74	54	87	58	85	62	103	70	107	69	113	71
		1	89	69	8	2	8	2	7	1	10	2						
0.70	0.20	2	89	69	14	4	14	4	17	3	19	4						
		4	89	69	17	3	20	9	21	8	22	12	25	11	24	10	25	15
		1	187	126	17	2	19	5	20	4	19	5						
0.70	0.40	2	187	126	26	6	28	6	31	7	28	5						
		4	187	126	32	16	35	18	42	20	44	22	49	21	51	21	55	26
		1	275	226	16	4	21	6	22	6	25	7						
0.70	0.60	2	275	226	39	14	34	15	37	19	39	20						
		4	275	226	41	23	53	31	55	34	65	36	59	32	67	35	73	34
		1	375	287	21	12	28	13	26	15	30	16						
0.70	0.80	2	375	287	33	18	40	23	40	26	48	24						
		4	375	287	52	32	70	39	78	45	77	48						

is of importance as revealed in Table 4. The precision of estimation might be affected by the variation of the host density but little by the activity of the parasite.

Acknowledgements

The author would like to acknowledge the fine assistance of Mr. Hsia In-shong in computation.

寄生蜂與田間害蟲相互交感之統計模型

林 燦 隆

本文提供決定寄生蜂在不同飛翔係數（公式 (1) 中之 λ）下的最佳釋放量與釋放地點數的一種統計模型與方法。這一模型能呈現經濟學上報酬遞減的法則及害蟲控制效果可因增加釋放地點數而增加的事實。若害蟲爲害作物之損失，飼養及釋放寄生蜂的費用均已知時，當可由預先妥爲設計之實際田間試驗估計模型中之未知介量並推測害蟲分布型，以算其最適當的釋放量與釋放地點數。

又由模擬試驗知取樣變異相當大，設計田間試驗時，必須愼重。

References

BARTLETT, M. S. (1960) Stochastic Population Models in Ecology and Epidemiology, pp. 36-37, Methuen & Co.

BAILEY, N. T. J. (1963) The Elements of Stochastic Processes. pp. 186. John Wiley.

MORAN, P. A. P. (1950) Some remarks on animal population dynamics. Biometrics Vol. 6, No. 3, pp. 250-258.

PEARCE, E. C. (1970) A new deterministic model for the interaction between predator and Prey. Biometrics Vol. 26, No. 3, 387-392.

PIELOU, E. C. (1970) An Introduction to Mathematical Ecology. pp. 70-71. John Wiley & Sones.

SAMUELSON, P. (1971) Generalized predator-prey oscillations in ecological and economic equilibrium. Proc. Nat. Acad. Sci. U. S. A. Vol. 68, No. 5, pp. 980-983.