Bot. Bull. Academia Sinica **16**: 177-190 (1975)

DISTRIBUTION OF THE EGG CLUSTERS OF *SCIRPOPHAGA NIVELLA* (LEPIDOPTERA: PYRALIDAE) ON SUGAR CANE[1]

TSAN-LONG LIN and TSEN LIN

Department of Agronomy, National Taiwan University
Taipei, Taiwan, Republic of China

(Recieved for publication January 29, 1975)

Abstract

A two-parameter Poisson process with mean function $\alpha(1-\beta^{t})(0<\beta<1)$ was used to describe and to predict the distribution of the egg clusters of ***Scirpophaga nivella*** on sugar cane. The maximum likelihood method (MLM) was used to estimate the unknown parameters β and ξ. Estimates of β with the MLM and Bayesian are compared. With tables provided, the maximum likelihood estimate can be obtained easily. Uniqueness of the estimates is obtained. The sampling behaviors are studied by using the Monte Carlo simulation. From the results of the latter, a method to determine the optimal observation size is described.

Introduction

The larva of *Scirpophaga nivella*, an important sugar cane pest, bores through young shoots of sugar cane soon after it hatches out of the egg and causes dead-heart. Since the degree of damage caused by the pest is closely related to the density of egg clusters (Lin, Liang and Lin, 1972), the knowledge of the distribution pattern of the egg clusters and the change of the egg clusters at some future time are considerably important.

Lin *et al.* (1972) pointed out that the Poisson process with a properly chosen mean function might adequately be used to describe the distribution of the egg cluster of *S. nivella* and its change. In the present paper, the Poisson process with two-parameter mean function was used to analyze the data which obtained from Hsin-Ying Sugar Cane Improvement Station. The maximum likelihood method was used to estimate the unknown parameters. Uniqueness of the estimates is also obtained.

Monte Carlo simulation was used to discuss the distribution of the estimates

(1) This work was supported by the National Science Council, Rep. of China.

and to determine the optimum number of plots. The possibility of using the estimated process to forecast the change of distribution is also discussed.

Distribution

Sampling of egg clusters was made at Hsin-Ying Sugar Cane Improvement Station, Taiwan Sugar Cooperation during the crop year of 1967 to 1968 by Mr. Liang of Taiwan Sugar Research Institute. An area was taken for investigation started on the 26th of september 1967 at three-week interval. The area was divided into 15 blocks, and each block was subdivided into 40 plots with a size of 4×1.25 meter square of the plot.

The cumulative number of egg clusters up to the time of observations are used to obtain means, mean squares, ratios of mean square/mean and $\sum(x_i-\bar{x})^2/\bar{x}$. Results are shown in Table 1. Since $\sum(x_i-\bar{x})^2/\bar{x}$ takes the well known form of $\sum(O-E)^2/E$, it approximately follows the chi-square distribution with 39 degrees of freedom. The values of 10% and 5% points of $\chi^2_{(39)}$ are 50.6 and 54.3, respectively, thus the distribution of cumulative number of *S. nivella* egg clusters up to time t in a plot may be adequately followed by a Poisson distribution such as

$$e^{-\Psi(t)}[\Psi(t)]^x/x!$$

where $\Psi(t)$ is a properly chosen function of time.

To obtain a proper form of mean function, the observed mean from 40 plots in each block was plotted against time, thus the results were shown (Fig. 1). When the growth rate of egg clusters reached a peak, it decreased and tended to zero. The results indicated that the curves of the mean function approached an asymptote sharply. Therefore, if the instaneous growth rate of egg clusters is assumed to be an exponential form, the mean function can be evaluated from

$$\Psi(t)=\int_0^t ae^{-bz}\,dz=\frac{a}{b}(1-e^{-bt})=\alpha(1-\beta^t),\qquad 0<\beta<1.$$

Thus, the probability of observing egg clusters x in the time interval of $(0, t)$ may be taken as

$$P_x(t)=e^{-\alpha(1-\beta^t)}[\alpha(1-\beta^t)]^x/x!,\qquad 0<\beta<1 \tag{1}$$

and the probability of egg clusters Y newly observed in the time interval of $(t-1, t)$ is given by

$$\begin{aligned} g_Y(t) &= e^{-\alpha(1-\beta)\beta^{t-1}}[\alpha(1-\beta)\beta^{t-1}]^Y/Y! \\ &= e^{-\xi\beta^t}(\xi\beta^t)^Y/Y! \end{aligned} \tag{2}$$

Table 1. *Statistical analyses of egg clusters in each block.*

Time of observation (t)	Item	Blocks														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Mean	0.250	0.200	0.050	0.100	0.150	0.100	0.100	0.175	0.125	0.175	0.125	0.200	0.125	0.175	0.350
	Mean square	0.192	0.164	0.049	0.092	0.131	0.092	0.144	0.199	0.112	0.199	0.163	0.215	0.163	0.148	0.438
	Mean square/Mean	0.769	0.821	0.974	0.923	0.972	0.923	1.436	1.139	0.897	1.139	1.308	1.077	1.308	0.846	1.253
	$\sum(x_i-\bar{x})^2/\bar{x}$	30.000	32.000	38.000	36.000	34.000	36.000	56.000	44.429	35.000	44.429	51.000	42.000	51.000	33.000	48.857
2	Mean	0.400	0.350	0.225	0.200	0.225	0.300	0.125	0.250	0.150	0.250	0.200	0.250	0.150	0.325	0.425
	Mean square	0.451	0.285	0.281	0.164	0.179	0.267	0.163	0.244	0.131	0.295	0.267	0.244	0.182	0.328	0.558
	Mean square/Mean	1.128	0.813	1.251	0.821	0.795	0.889	1.308	0.974	0.872	1.179	1.333	0.974	1.214	1.008	1.314
	$\sum(x_i-\bar{x})^2/\bar{x}$	44.000	31.714	48.778	32.000	31.000	34.667	51.000	38.000	34.000	46.000	52.000	38.000	47.333	39.308	51.235
3	Mean	0.450	0.425	0.325	0.200	0.225	0.300	0.150	0.250	0.175	0.300	0.300	0.275	0.150	0.350	0.425
	Mean square	0.459	0.353	0.328	0.164	0.179	0.267	0.182	0.244	0.148	0.421	0.369	0.256	0.182	0 336	0.558
	Mean square/Mean	1.020	0.831	1.008	0.821	0.795	0.889	1.214	0.974	0.846	1.402	1.231	0.930	1.214	0.960	1.314
	$\sum(x_i-\bar{x})^2/\bar{x}$	39.778	32.412	39.308	32.000	31.000	34.667	47.333	38.000	33.000	54.667	48.000	36.273	47.333	37.429	51.235
4	Mean	0.450	0.425	0.325	0.225	0.225	0.350	0.150	0.275	0.175	0.325	0.325	0.275	0.150	0.350	0.425
	Mean square	0.459	0.353	0.328	0.179	0.179	0.285	0.182	0.256	0.148	0.430	0.430	0.256	0.182	0.336	0.558
	Mean square/Mean	1.020	0.831	1.008	0.795	0.795	0.813	1.214	0.930	0.846	1.323	1.323	0.930	1.214	0.960	1.314
	$\sum(x_i-\bar{x})^2/\bar{x}$	39.778	32.412	39.308	31.000	31.000	31.714	47.333	36.273	33.000	51.615	51.615	36.273	47.333	37.429	51.235
5	Mean	0.450	0.450	0.325	0.250	0.225	0.350	0.150	0.275	0.175	0.325	0.325	0.275	0.150	0.375	0.425
	Mean square	0.459	0.356	0.328	0.192	0.179	0.285	0.182	0.256	0.148	0.430	0.430	0.256	0.182	0.343	0.558
	Mean square/Mean	1.020	0.792	1.008	0.769	0.795	0.813	1.214	0.930	0.846	1.323	1.323	0.930	1.214	0.915	1.314
	$\sum(x_i-\bar{x})^2/\bar{x}$	39.778	30.889	39.308	30.000	31.000	31.714	47.333	36.273	33.000	51.615	51.615	36.273	47.333	35.667	51.235
6	Mean	0.450	0.450	0.325	0.250	0.225	0.350	0.150	0.275	0.200	0.325	0.325	0.275	0.150	0.375	0.425
	Mean square	0.459	0.356	0.328	0.192	0.179	0.285	0.182	0.256	0.164	0.430	0.430	0.256	0.182	0.343	0.558
	Mean square/Mean	1.020	0.792	1.008	0.769	0.795	0.813	1.214	0.930	0.821	1.323	1.323	0.930	1.214	0.915	1.314
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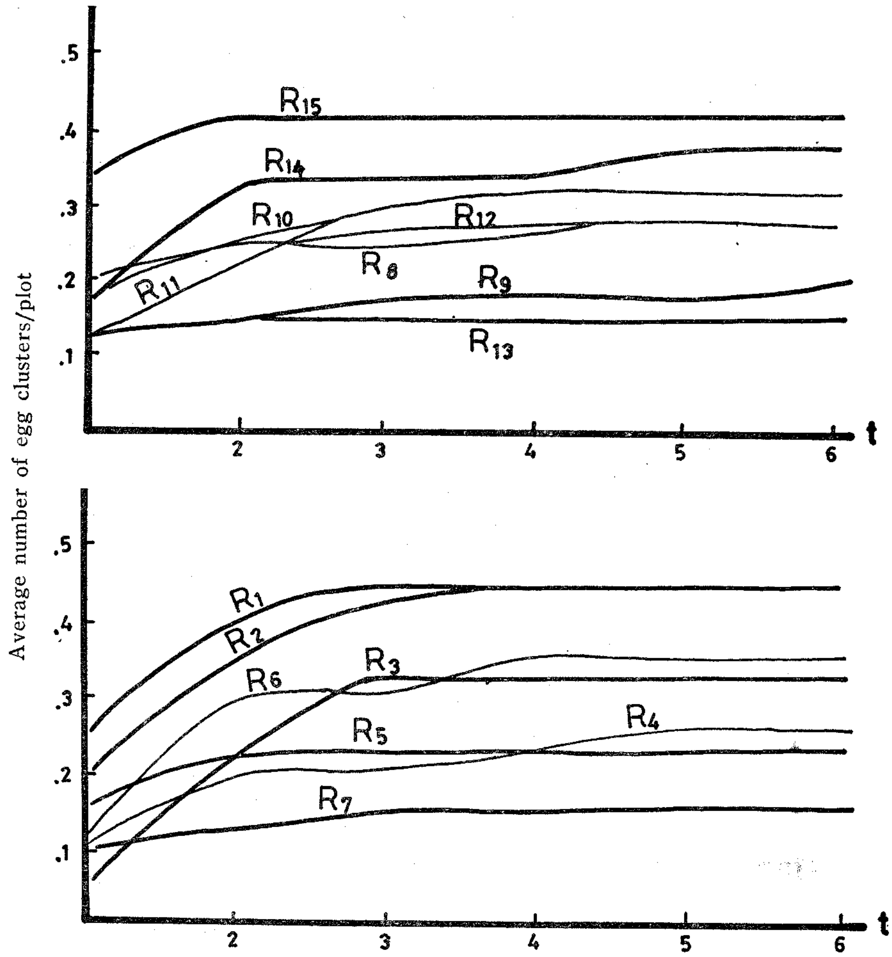

Fig. 1. The curves of observed mean function of egg clusters in each block of 40 plots at time t. (R_i=the ith block, t_1=Sept. 26, 1967)

where

$$\xi = \frac{\alpha(1-\beta)}{\beta} \tag{3}$$

Estimation

If observations are made at m equispaced time, at the same n plots of each m, time t may be coded into $t=1, 2, \ldots, m$. If $\{y_{ij}\}$ are used to denote the number of egg clusters newly observed at the jth plot in interval $(i-1, i)$, the likelihood function will be given by

$$L(y_{11}, y_{12}, \ldots, y_{1n}, y_{21}, \ldots, y_{mn}) = \exp\left(-n\xi \sum_{i=1}^{m} \beta^i\right) \prod_{i=1}^{m} (\xi\beta^i)^{y_{i\cdot}} \Big/ \prod_{i=1}^{m} \prod_{j=1}^{n} y_{ij}! \tag{4}$$

where $y_{i\cdot}=\sum_{j=1}^{n} y_{ij}$. If the log-likelihood function of (4) is differentiated with respect to ξ and β, we obtain

$$\left.\begin{aligned}\frac{\partial \log L}{\partial \xi}&=-n\sum_{i=1}^{m}\beta^{i}+y_{\cdot\cdot}/\xi\\ \frac{\partial \log L}{\partial \beta}&=-\xi n\sum_{i=1}^{m} i\,\beta^{i-1}+\sum_{i=1}^{m} i\,y_{i\cdot}/\beta\end{aligned}\right\}\qquad(5)$$

where $y_{\cdot\cdot}=\sum_{i=1}^{m} y_{i\cdot}=\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij}$. By setting both of (5) equal to zero, and eliminated ξ, we obtained;

$$\frac{\sum i\,y_{i\cdot}}{y_{\cdot\cdot}}-\frac{1}{1-\beta}+\frac{m\beta^{m}}{1-\beta^{m}}=0\qquad(6)$$

then

$$\xi=\frac{y_{\cdot\cdot}(1-\beta)}{n\beta(1-\beta^{m})}\ .\qquad(7)$$

Since $0<\beta<1$, for sufficiently large value of m, the third term of (6) becomes negligibly small, the value

$$\beta^{(0)}=1-1/A$$

where $A=\sum_{i=1}^{m} i\,y_{i\cdot}/y_{\cdot\cdot}$, may be used as an initial value in the iterative process. Also, for $0<\beta<1$,

$$\frac{1}{1-\beta}-\frac{m\beta^{m}}{1-\beta^{m}}\qquad(8)$$

is a monotonic increasing function of β, then a unique solution satisfying (5) can be found in

$$1>\beta>1-1/A\qquad(9)$$

Since

$$\left.\begin{aligned}\frac{\partial^2 \log L}{\partial \xi^2}&=-y_{\cdot\cdot}/\xi^2\\ \frac{\partial^2 \log L}{\partial \xi\,\partial \beta}&=\frac{\partial^2 \log L}{\partial \beta\,\partial \xi}=-n\sum_{i=1}^{m} i\,\beta^{i-1}\\ \frac{\partial^2 \log L}{\partial \beta^2}&=-n\xi\sum_{i=1}^{m} i(i-1)\,\beta^{i-2}-\sum_{i=1}^{m} i\,y_{i\cdot}/\beta^2\end{aligned}\right\}\qquad(10)$$

the information matrix is given by

$$I=\begin{bmatrix} n\sum_{i=1}^{m}\beta^{i}/\xi & n\sum_{i=1}^{m} i\,\beta^{i-1}\\ n\sum_{i=1}^{m} i\,\beta^{i-1} & n\xi\sum_{i=1}^{m} i^2\,\beta^{i-2}\end{bmatrix}$$

then

$$|I| = n^2 \sum_{i=1}^{m-1} \sum_{j>i}^{m} (j-i)^2 \beta^{i+j-2} \tag{11}$$

Also, from (8), Hessian matrix is given by

$$\begin{bmatrix} -y_{..}/\xi^2 & -n \sum_{i=1}^{m} i\, \beta^{i-1} \\ -n \sum_{i=1}^{m} i\, \beta^{i-1} & -n\xi \sum_{i=1}^{m} i(i-1)\, \beta^{i-2} - \sum_{i=1}^{m} i\, y_{i.}/\beta^2 \end{bmatrix}$$

From the values of ξ and β with the satisfaction of (6) and (7), the determinant of Hessian is

$$n^2 \sum_{i=1}^{m} \sum_{j>i}^{m} (j-i)^2 \beta^{i+j-2} > 0$$

Therefore, from (10) and the monotonisity of (8), a unique maximum likelihood estimate (MLE) of β can be found in the interval of (9) and hence to estimate of ξ.

As the method described above, the following results are obtained from the observations at the Hsin-Ying Sugar Cane Improvement Station:

$y_{i.} = 96, 57, 19, 6, 3, 1$

$A = \sum i\, y_{i.}/y_{..} = 1.7143$

$\xi = 0.4059$

$\beta = 0.4292$

$\alpha = 0.3052$

Tables of $1/(1-\beta) - m\beta^m/(1-\beta^m)$, $1.0/(1-\beta^m)$ and $(1-\beta)/(\beta(1-\beta^m))$ are provided in Appendix (I–III) for simplifying the estimation procedure with a desk calculator, if the digital computer is not available.

In Bayesian approach, the Jeffrey's prior is defined by

$$P(\xi, \beta) \propto |I|^{1/2}$$

and with this non-informative prior, the joint postorior is given by

$$P(\xi, \beta | y) \propto |I|^{1/2} e^{-n\xi \sum_{i=1}^{m} \beta^i} \prod_{i=1}^{m} (\xi\beta^i)^{y_{i.}} \tag{12}$$

Integrating ξ out, we obtain

$$P(\beta | y) \propto |I|^{1/2} \int_0^\infty e^{-n\xi \sum_{i=1}^{m} \beta^i} \xi^{y_{i.}} \beta^{\sum_{i=1}^{m} i y_{i.}} d\xi$$

$$\propto |I|^{1/2} \beta^{\sum_{i=1}^{m} i y_{i.}} \Big/ \left(\sum_{i=1}^{m} \beta^i\right)^{y_{..}+1}$$

or

$$P(\beta | y) \propto \left[\sum_{i=1}^{m-1} \sum_{j>i}^{m} (j-i)^2 \beta^{i+j-2}\right]^{1/2} \beta^{\sum_{i=1}^{m} i y_{i.} - y_{..} - 1} \left[\frac{1-\beta}{1-\beta^m}\right]^{y_{..}+1} \tag{13}$$

For $m=6$, as in the observation at Hsin-Ying Sugar Cane Improvement Station,

$$P(\beta|y) \propto \beta^{\sum_{i=1}^{m} i\,y_{i\cdot} - y_{\cdot\cdot} - 1} \left[\frac{1-\beta}{1-\beta^m}\right]^{y_{\cdot\cdot}+1}$$

$$\cdot\,[\beta+4\beta^2+10\beta^3+20\beta^4+35\beta^5+20\beta^6+10\beta^7+4\beta^8+\beta^9]^{1/2}$$

and for the observed values of $y_{\cdot\cdot}=182$ and $\sum_{i=1}^{m} i\,y_{i\cdot}=312$

$$P(\beta|y) \propto \frac{|I|^{1/2}\,\beta^{129}\,(1-\beta)^{183}}{(1-\beta^6)^{183}}$$

The numerical intergration is involved to evaluate the expectation and the distribution function of β. The results are shown below Table 2.

Table 2.

β	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$P(\beta\|\cdot)$	0.0^7	0.0^7	0.0004468	8.9125925	1.0869448	0.0000158	0.0^7
$\int_0^\beta P(U\|\cdot)dU$	0.0^7	0.0^7	0.0000223	0.4456743	0.9456512	0.9999992	1.0

The expectation is 0.4609 which is very close to the maximum likelihood estimate. The mode of the posterior distribution of β appears between 0.42–0.44. Thus, the shortest of 90% and 95% intervals are (0.37, 0.47) and (0.36, 0.48) respectively (Fig. 2).

The Empirical Distribution of the Estimates

To study the behavior of the estimates, Monte Carlo simulations were used to obtain the various values of ξ, β and n. Among which only the results of $\beta=0.4$, $\xi=0.4$ were fitted to the observations at Hsin-Ying Sugar Cane Improvement Station. Those results are shown in Appendix IV when $m=6$. The congruence method of $x_{n+1}=23x_n \pmod{10^8+1}$ with initial value $x_0 = 47{,}594{,}118$ was used to generate uniform pseudo-random numbers. The method described by Abramowitz and Stegun (1965) was used to generate the required Poisson series. From each combination of ξ, β and n, one hundred sets of data are generated, and each set of data, the MLE of ξ and β are calculated.

From the combination of ξ, β and n ($\xi=0.4$, 0.5, 0.6, $\beta=0.4$, 0.5, 0.6 and $n=10, 20, \ldots, 100$) only half of the estimated pairs (ξ, β) lie within the interval [$\beta\pm0.1$, $\xi\pm0.1$], and the situations are much worse for samples of the smaller size. If the interest is in the rate of increase, β, the sample size of $n=100$ is enough to obtain 90% of the estimate values within the interval of $\beta\pm0.1$,

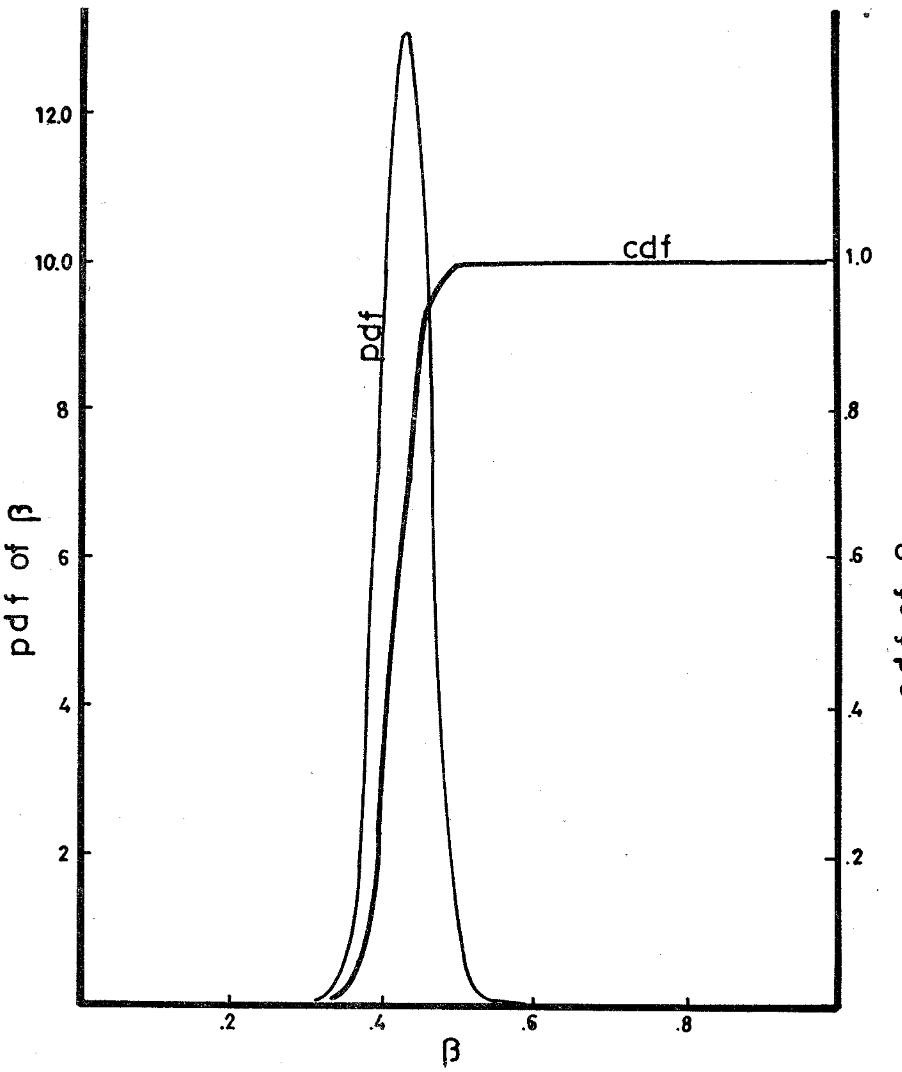

Fig. 2. The pdf and cdf of β

but for estimates of ξ, the same size of sample gives only 50% of the estimates to be included within the interval of $\xi \pm 0.1$.

When n is small, the probability of observing zero counts from $t \geq 0$ to $t=m$ is high. Hence, no estimates of β and ξ are available.

The optimum size of sample depends on the cost of the additional observations to be made, and the loss is due to discrepancy between parameters and the estimates. The late or early growth of the crop will be hindered by the overestimate or underestimate of β. Because, according to the overestimates of β, the pest control action will be done too early. If the estimate of β is within the limits of $\beta \pm 0.1$, the optimal sample size may be determined from the following formula as

$$k(n) = C \cdot n + P^{L}_{n} r_{L} + P^{U}_{n} r_{U}$$

where C is the cost of observing an additional plot, P^{L}_{n} and P^{U}_{n} are the probabilities of obtaining under- and overestimates, and r_L and r_U are corresponding regrets. We selected n to minimize $k(n)$ that was described in (12). The pooled probability of the estimate β within $\beta \pm 0.1$, obtained from the simulation over the range of values $0.4 \leq \beta \leq 0.6$ and $0.4 \leq \xi \leq 0.6$, are given as below Table 3.

Table 3.

Sample size	10	20	30	40	50	60	70	80	90	100
Probability	0.35	0.40	0.62	0.68	0.73	0.78	0.82	0.86	0.88	0.89

For convenience, putting $r_U = r_L = r = aC$, a is the cost ratio, we obtained the values of $k(n)/C$ in Table 4.

Table 4. *The values of $k(n)/C$ for $r_U = r_L = r = aC$.*

a \ n	10	20	30	40	50	60	70	80	90	100
20	23.0	32.0	37.6	46.4	55.4	64.4	73.8	82.8	92.4	102.4
40	36.0	44.0	45.2	52.8	60.8	68.8	77.6	85.6	94.8	104.4
60	49.0	56.0	55.8	59.2	66.2	73.2	81.4	88.4	98.4	106.6
80	62.0	68.0	60.4	65.6	71.6	77.6	85.2	91.2	99.6	108.8
100	75.0	80.0	68.4	74.0	77.0	82.0	89.0	94.0	102.0	111.0
150	107.5	110.0	87.0	91.0	90.5	93.0	98 5	101.0	108.0	116.5
200	140.0	140.0	106.0	104.0	104.0	104.0	104.0	108.0	114.0	122.0

In these examples, we found that the optimum size is $\hat{n} = 10$ for $a \leq 60$, $\hat{n} = 30$ for $80 \leq a \leq 150$ and $\hat{n} = 40$ for $a = 200$.

Forecasting

Investigation at Hsin-Yin Sugar Cane Improvement Station described earlier is attemped to discuss the probability of forecasting the distribution of egg clusters. The results are summerized in the Table 5.

From Table 5, it was found that the forecasting based on observations of only two time points (i.e. at 26th September and 17th October) was enough to predict the future distributions for this case. This result is mainly based on the nature of mean function of the process as well as the nature of the

Table 5A. *Forecasting S. nivella egg clusters in sugar cane by using a Poisson process.*

Predicted numbers of increase in $(t-1, t)$.

Time of forecast	Epoch forecasted	Mean rate of increase ($\bar{y}_{i\cdot}$)		Frequency								
				Observed			Predicted with $e^{-\xi\beta^t}(\xi\beta)^x/x!$			Predicted with $e^{-\bar{y}_{i\cdot}}(y_{i\cdot})^n/n!$		
		Observed	Predicted	0	1	2	0	1	2	0	1	2
2	2	0.10	0.09	548	47	5	545.6	51.8	2.5	545.62	51.83	2.46
	3	0.03	0.06	582	17	1	567.1	32.0	0.9	581.3	18.41	0.29
	4	0.01	0.03	594	6		580.24	19.43		594.0	5.9	
	5	0.00	0.02	597	3		588.2	11.7		597.0	3.0	
	6	0.00	0.01	599	1		593.0	7.0		599.0	1.0	
3	3	0.03	0.04	582	17	1	577.1	22.4	0.4	581.3	18.4	
	4	0.01	0.02	594	6		588.9	11.0		594.0	6.0	
	5	0.00	0.01	597	3		594.6	5.37		597.0	3.0	
	6	0.00	0.00	599	1		597.39	2.6		599.0	1.0	
4	4	0.01	0.01	594	6		591.1	8.82		594.0	5.9	
	5	0.00	0.01	597	3		596.0	3.94		597.0	3.0	
	6	0.00	0.00	599	1		598.2	1.75		599.0	1.0	
5	5	0.00	0.01	597	3		596.3	3.7		597.0	3.0	
	6	0.00	0.00	599	1		598.4	1.61		599.0	1.0	
6	6	0.00	0.00	599	1		598.4	1.61		599.0	1.0	

Table 5B. *Forecasting S. nivella egg clusters in sugar cane by using a Poisson Process.*

Predicted number of egg clusters in $(0, t)$.

Time of forecast	Epoch forecasted	Mean		Frequency								Goodness of fit χ^2	Frequency calculated with respective means				Goodness of fit χ^2
				Observed				Predicted									
		Predicted	Observed	0	1	2	3	0	1	2	3		0	1	2	3	
2	2	0.26	0.25	469	111	18	2	464.95	118.56	15.12	1.28	1.2677	464.95	118.56	15.12	1.28	1.2677
	3	0.31	0.29	453	125	19	3	439.45	136.85	21.31	2.21	2.0156	450.46	129.13	18.51	1.77	0.6689
	4	0.34	0.30	448	129	20	3	424.98	146.57	25.28	2.91	4.9721	445.98	132.31	19.63	1.94	0.4749
	5	0.36	0.30	445	132	20	3	416.61	151.97	27.72	3.37	7.8575	443.75	133.86	20.19	2.03	0.3451
	6	0.38	0.30	444	133	20	3	411.72	155.05	29.19	3.66	10.3771	443.01	134.38	20.38	2.06	0.3178
3	3	0.29	0.29	455	125	19	3	450.46	129.13	18.15	1.77	0.6689	450.46	129.13	18.51	1.77	0.6689
	4	0.31	0.30	448	129	20	3	442.10	135.02	20.62	2.10	0.6482	445.98	132.31	19.63	1.94	0.4749
	5	0.31	0.30	445	132	20	3	438.12	137.76	21.66	2.27	0.6729	443.75	133.86	20.19	2.03	0.3451
	6	0.32	0.30	444	133	20	3	436.22	139.06	22.17	2.36	0.7858	443.01	134.38	20.38	2.06	0.3178
4	4	0.30	0.30	448	129	20	3	445.98	132.31	19.63	1.94	0.4749	445.98	132.31	19.63	1.94	0.4749
	5	0.30	0.30	445	132	20	3	443.04	134.36	20.37	2.06	0.3527	443.75	133.86	20.19	2.03	0.3451
	6	0.31	0.30	444	133	20	3	441.74	135.26	20.71	2.11	0.3369	443.01	134.38	20.38	2.06	0.3178
5	5	0.30	0.30	445	132	20	3	443.75	133.86	20.19	2.03	0.3451	443.75	133.86	20.19	2.03	0.3451
	6	0.30	0.30	444	133	20	3	442.56	134.70	20.50	2.07	0.3211	443.01	134.38	20.38	2.06	0.3178
6	6	0.30	0.30	444	133	20	3	443.01	134.38	20.38	2.06	0.3178	443.01	134.38	20.38	2.06	0.3178

pest.

Therefore, if the growth of the pest as affected by the environmental factors was normal in 1967 and 1968, the control action of pest growth after October would be unnecessary.

Literature Cited

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甘蔗白螟卵塊的分布型

林燦隆　林　貞

國立臺灣大學農藝學系

白螟爲甘蔗之主要害蟲，其幼蟲鑽進蔗莖而造成枯心，致使甘蔗產量降低。因此，爲達成有效的防治白螟幼蟲，須知白螟幼蟲之動態，然幼蟲動態在其危害之前不易把握，故由卵塊之多寡推測幼蟲期蟲數的變動情形較爲方便。

本文係利用平均函數爲

$$\Psi(t)=\int_0^t ae^{-bz}\,dz=\frac{a}{b}(1-e^{-bt})=\alpha(1-\beta^t),\qquad 0<\beta<1$$

的二介量卜瓦松過程探求白螟卵塊的分布情形。故在 $(t-1,\ t)$ 時間內觀察到 Y 個卵塊的機率爲

$$g_Y(t)=e^{-\xi\beta^t}(\xi\beta^t)^Y/Y!,\qquad \xi=\frac{\alpha(1-\beta)}{\beta}$$

根據新營糖業改良場所調查白螟卵塊資料，求得上式中二未知介量 β、ξ 的最大可能估值分別爲 0.4292、0.4059，再以貝氏法求得 β 估值爲 0.4609。由二法所求得估值極近似，但最大可能法在計算上較簡單。由貝氏法求得 β 之 90% 及 95% 最短間隔分別爲 (0.37、0.47) 與 (0.36、0.48)。

用費用函數 $k(n)=\mathrm{C}\cdot n+\mathrm{P}_n^{\mathrm{L}}r_{\mathrm{L}}+\mathrm{P}_n^{\mathrm{U}}r_{\mathrm{U}}$，令 $r_{\mathrm{U}}=r_{\mathrm{L}}=r=a\mathrm{C}$，求得上述資料中在不同 a 值的最適取樣數 $\hat{n}$ 時，知 $a\leq 60$ 時 $\hat{n}=10$；$80\leq a\leq 150$，$\hat{n}=30$，且 $a=200$ 時，$\hat{n}=40$。

同時根據分析結果，由新營場56年 9 月26日及10月17日兩次的調查資料可預測未來卵塊的分布情形，故若56～57年的環境因子對白螟生長的影響爲正常，則在10月以後的防治工作似乎是不需要的。

Appendix I: *Table of* $1/(1-\beta)-m\beta^m/(1-\beta^m)$.

β \ m	2	3	4	5	6	7	8	9	10
0.10	1.090909	1.108108	1.110711	1.111061	1.111105	1.111110	1.111111	1.111111	1.111111
0.15	1.130435	1.166311	1.174445	1.176091	1.176402	1.176459	1.176469	1.176470	1.176471
0.20	1.166667	1.225806	1.243590	1.248399	1.249616	1.249910	1.249980	1.249995	1.249999
0.25	1.200000	1.285714	1.317647	1.328446	1.331868	1.332906	1.333211	1.333299	1.333324
0.30	1.230769	1.345324	1.395907	1.416392	1.424194	1.427040	1.428047	1.428394	1.428512
0.35	1.259259	1.404075	1.477522	1.512062	1.527412	1.533955	1.536660	1.537752	1.538186
0.40	1.285714	1.461538	1.561576	1.614937	1.641990	1.655179	1.661420	1.664307	1.665618
0.45	1.310345	1.517398	1.647143	1.724183	1.767942	1.791927	1.804707	1.811367	1.814776
0.50	1.333333	1.571429	1.733333	1.838710	1.904762	1.944882	1.968627	1.982387	1.990225
0.55	1.354839	1.623482	1.819330	1.957244	2.051410	2.114004	2.154669	2.180582	2.196828
0.60	1.375000	1.673469	1.904412	2.078418	2.206364	2.298401	2.363335	2.408377	2.439166
0.65	1.393939	1.721351	1.987964	2.200848	2.367718	2.496297	2.593837	2.666793	2.720678
0.70	1.411765	1.767123	2.069483	2.323212	2.533318	2.705117	2.843937	2.954879	3.042647
0.75	1.428571	1.810811	2.148571	2.444302	2.700921	2.921674	3.109996	3.269379	3.403260
0.80	1.444444	1.852459	2.224932	2.563065	2.868332	3.142434	3.387248	3.604777	3.797098
0.85	1.459459	1.892128	2.298356	2.678625	3.033536	3.363805	3.670246	3.953759	4.215314
0.90	1.473684	1.929889	2.368712	2.790286	3.194782	3.582407	3.953399	4.308030	4.646601

Appendix II: *Table of* $1.0/(1-\beta^m)$.

β \ m	2	3	4	5	6	7	8	9	10
0.10	1.010101	1.001001	1.000100	1.000010	1.000001	1.000000	1.000000	1.000000	1.000000
0.15	1.023018	1.003386	1.000507	0.000076	1.000011	1.000002	1.000000	1.000000	1.000000
0.20	1.041667	1.008065	1.001603	1.000320	1.000064	1.000013	1.000003	1.000001	1.000000
0.25	1.066667	1.015873	1.003922	1.000978	1.000244	1.000061	1.000015	1.000004	1.000001
0.30	1.098901	1.027749	1.008166	1.002436	1.000730	1.000219	0.000066	1.000020	1.000006
0.35	1.139601	1.044796	1.015235	1.005280	1.001842	1.000644	1.000225	1.000079	1.000028
0.40	1.190476	1.068376	1.026273	1.010346	1.004113	1.001641	1.000656	1.000262	1.000105
0.45	1.253918	1.100261	1.042760	1.018800	1.008373	1.003751	1.001684	1.000757	1.000341
0.50	1.333333	1.142857	1.066667	1.032258	1.015873	1.007874	1.003922	1.001957	1.000978
0.55	1.433692	1.199580	1.100723	1.052996	1.028469	1.015460	1.008444	1.004627	1.002539
0.60	1.562500	1.275510	1.148897	1.084316	1.048939	1.028800	1.017083	1.010180	1.006083
0.65	1.731602	1.378597	1.217295	1.131259	1.081571	1.051549	1.032913	1.021150	1.013646
0.70	1.960784	1.522070	1.315963	1.202024	1.133336	1.089745	1.061175	1.042050	1.029069
0.75	2.285714	1.729730	1.462857	1.311140	1.216513	1.154047	1.111251	1.081181	1.059674
0.80	2.777778	2.049180	1.693767	1.487387	1.355278	1.265367	1.201594	1.155025	1.120290
0.85	3.603604	2.591513	2.092078	1.797608	1.605522	1.471837	1.374553	1.301434	1.245135
0.90	5.263158	3.690037	2.907822	2.441943	2.134203	1.916799	1.755825	1.632441	1.535340

Appendix III: *Table of* $(1.0-\beta)/\beta(1-\beta^m)$.

β \ m	2	3	4	5	6	7	8	9	10
0.10	9.090909	9.009009	9.000900	9.000090	9.000009	9.000001	9.000000	9.000000	9.000000
0.15	5.797101	5.685856	5.669537	5.667097	5.666731	5.666676	5.666668	5.666667	5.666667
0.20	4.166667	4.032258	4.006410	4.001280	4.000256	4.000051	4.000010	4.000002	4.000000
0.25	3.200000	3.047619	3.011765	3.002933	3.000733	3.000183	3.000046	3.000011	3.000003
0.30	2.564103	2.398082	2.352388	2.339017	2.335036	2.333844	2.333486	2.333379	2.333347
0.35	2.116402	1.940335	1.885436	1.866948	1.860563	1.858338	1.857561	1.857289	1.857194
0.40	1.785714	1.602564	1.539409	1.515519	1.506169	1.502462	1.500984	1.500393	1.500157
0.45	1.532567	1.344764	1.274484	1.245200	1.232456	1.226806	1.224281	1.223148	1.222639
0.50	1.333333	1.142857	1.066667	1.032258	1.015873	1.007874	1.003922	1.001957	1.000978
0.55	1.173021	0.981475	0.900592	0.861542	0.841474	0.830831	0.825091	0.821967	0.820259
0.60	1.041667	0.850340	0.765931	0.722878	0.699293	0.685867	0.678055	0.673454	0.670722
0.65	0.932401	0.742322	0.655466	0.609139	0.582384	0.566219	0.556184	0.549850	0.545810
0.70	0.840336	0.652316	0.563984	0.515153	0.485715	0.467034	0.454789	0.446593	0.441029
0.75	0.761905	0.576577	0.487619	0.437047	0.405504	0.384682	0.370417	0.360393	0.353225
0.80	0.694444	0.512295	0.423442	0.371847	0.338819	0.316342	0.300399	0.288756	0.280073
0.85	0.635930	0.457326	0.369190	0.317225	0.283327	0.259736	0.242568	0.229665	0.219730
0.90	0.584795	0.410004	0.323091	0.271327	0.237134	0.212978	0.195092	0.181382	0.170593

Appendix IV. *Results of the Monte Carlo simulation.*

(β=.1~1.0, ξ=.3~.6, n=10~100 and m=6)

n	10			20			30			40			50			60			70			80			90			100		
β \ ξ	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6	.3~.4	.4~.5	.5~.6
.1~ .2																														
.2~ .3					1			3	2	2	2	1		2	2		3			1	1			2			6			
.3~ .4				4	3	2	2	3	9	6	8	6	6	9	4	7	14	8	6	7	7	8	9	6	8	10	3	12	14	9
.4~ .5		1	6		6	1	11	5		7	2	3	14	9		8	6	2	12	4	1	20	6	2	20	7	3	23	9	3
.5~ .6	1	1	1	4	1		2			2	1		1	1		2			1											
.6~ .7																														
.7~ .8																														
.8~ .9																														
.9~1.0																														